

The Multiplexing Gain of MIMO X -Channels with Partial Transmit Side-Information

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Abstract—THIS PAPER IS ELIGIBLE FOR THE STUDENT PAPER AWARD. In this paper, we obtain the scaling laws of the sum-rate capacity of a MIMO X -channel, a 2 independent sender, 2 independent receiver channel with messages from each transmitter to each receiver, at high signal to noise ratios (SNR). The X -channel has sparked recent interest in the context of cooperative networks and it encompasses the interference, multiple access, and broadcast channels as special cases. Here, we consider the case with partially cooperative transmitters in which *only* partial and asymmetric side-information is available at one of the transmitters. It is proved that when there are M antennas at all four nodes, the sum-rate scales like $2M \log \text{SNR}$ which is in sharp contrast to $\left[\lfloor \frac{4M}{3} \rfloor, \frac{4M}{3}\right] \log \text{SNR}$ for non-cooperative X -channels [14], [11]. This further proves that, in terms of sum-rate scaling at high SNR, partial side-information at one of the transmitters and full side-information at both transmitters are equivalent in the MIMO X -channel.

I. INTRODUCTION

Cooperation in wireless networks has sparked much recent interest in the research community. Having wireless nodes cooperate at the transmitting and/or receiving end, can, for example, improve rates, diversity, or power utilization. How much gain cooperation provides depends on a number of factors, including the signal to noise ratio (SNR) and the amount of side information at the transmitters. In this work, we look at the value of partial transmitter cooperation, in terms of the scaling law of the sum-rate in the MIMO X -channel at high SNR.¹

The multiplexing gain of the sum-rate of the MIMO X -channel has been recently studied [14], [11], [12]. The MIMO X -channel is a simple 2 transmitter (2 Tx), 2 receiver (2 Rx) channel in which each Tx has a message for each Rx. Its study is of theoretical interest, as it encompasses classical channels such as the interference, the multiple access and the broadcast channels. While the classical MIMO X channel forbids cooperation between the two Tx, in this work we allow for transmitter cooperation, in the form of transmitter *side-information*.

The three channels shown in Fig. 1 illustrate three multiuser MIMO channels with different amounts of transmitter side-information. In Fig. 1(1), the MIMO X -channel

is illustrated². The two transmitters, Tx 1 and Tx 2 operate independently, and each wishes to send messages to each of the two non-cooperating receivers Rx 1 and Rx 2. The messages are numbered 11, 12, 21 and 22, with the convention that indices will follow the form (Tx# Rx#). We note that the MIMO interference channel is embedded in the MIMO X channel and may be obtained by eliminating the cross-over messages 12 and 21. The messages 11, 12, 21 and 22 will be simultaneously transmitted at the rates R_{11}, R_{12}, R_{21} and R_{22} respectively. The sum-rate achieved by this tuple is defined to be $R \triangleq R_{11} + R_{12} + R_{21} + R_{22}$.

In Fig. 1(2), the MIMO cognitive X -channel is depicted. We define this to be a channel in which Tx 2 knows message 11, but not message 12, of Tx 1. Tx 1 knows none of Tx 2's messages. We use the term *partial side-information* to refer to the fact that Tx 2 knows only one of Tx 1's messages, and the term *asymmetric side-information* to refer to the fact that Tx 2 has side-information regarding Tx 1 but not vice versa. We note that the side-information could have been both message 11 and 12 or just message 12 at Tx 2. In the latter event, analogous results are obtained by permuting the indices in our results. In the former, the rates achieved could be improved at low-medium SNR, but as we will see, the extra information contained in knowing the message 12 as well as message 11 is unnecessary as $\text{SNR} \rightarrow \infty$, where we will show that the partial information is sum-rate scaling optimal. In Fig. 1(3) shows a MIMO broadcast channel in which both Tx's share all their messages (full, symmetric side-information) [15], [1] with $2M$ transmit antennas and two independent Rx's with M antennas each.

The multiplexing gain of the channel depicted in Fig. 1(1) is defined as the limit of the ratio of the maximal achieved sum-rate R_i in the capacity region \mathcal{C}_i , to the $\log(\text{SNR})$ ³ as the $\text{SNR} \rightarrow \infty$, or

$$M_i = \lim_{\text{SNR} \rightarrow \infty} \frac{\max R_i(\text{SNR})}{\log(\text{SNR})}.$$

The multiplexing gain of the channel of Fig. 1(1) has been

¹Although the achievable rate region we derive is for general channels, the multiplexing gain results hold only for Gaussian noise channels, hence we will often omit the term Gaussian for brevity.

²Throughout this work the term MIMO will denote that there are M antennas at each of the four nodes Tx 1, Tx 2, Rx 1 and Rx 2. In order to simplify results we will sometimes set $M = 1$, in which case we drop the word MIMO.

³Note that the usual factor $\frac{1}{2}$ is omitted in any rate expressions, but rather the number of times the sum-rate looks like $\log(\text{SNR})$ is the multiplexing gain.

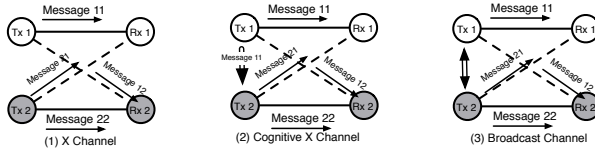


Fig. 1. Three 2×2 MIMO X -channels whose multiplexing gains are contrasted here. All Rxs decode independently. (1) MIMO X -channel with one message from each Tx to each Rx and no side-information. (2) MIMO cognitive X -channel with partial asymmetric side-information: Tx 2 knows message 11 but not message 12, and Tx 1 knows none of Tx 2's messages. (3) MIMO Broadcast channel: $2M$ transmitting antennas delivering messages to two independent M antenna Rxs.

recently studied [11], [14]. The MIMO interference channel embedded in channel (1) is known to achieve a multiplexing gain of M [12]. This channel has no cross-over messages (messages 12 and 21). Interestingly, when cross-over information is present, as in the MIMO X -channel of (1), the multiplexing gain lies in the range $[\lfloor \frac{4}{3}M \rfloor, \frac{4M}{3}]$ [11], [14], possibly improving upon the MIMO interference channel. The MIMO broadcast channel of Fig. 1(3) is known to have a multiplexing gain of $2M$, equal to the number of transmit, and receive antennas. In the broadcast channel, full, symmetric Tx side-information is used, that is, both Tx 1 and Tx 2 know *all* messages of the other. In this work, we show that such full transmit side-information is not strictly necessary to achieve the same sum-rate scaling of $2M$. We show that the partial side-information, shown in the MIMO cognitive X -channel also achieves a sum-rate scaling of $2M$ as $\text{SNR} \rightarrow \infty$.

In this work, we assume *non-causal* side-information, which may be either symmetric or asymmetric, but requires that all messages are known fully before transmission starts. This is in sharp contrast to the work [10], in which similar 2 Tx, 2 Rx channels with causal side-information are considered. In [10] it is shown that even if both the Txs and Rxs may cooperate using noisy links, in a causal fashion (although full duplex transmission is permitted) the multiplexing gain is always limited to 1 when $M = 1$. This is in sharp contrast to when non-causal side-information is present, resulting in a multiplexing gain of 2.

This paper is structured as follows. In section II we define the MIMO cognitive X channel and demonstrate an achievable rate region, which we to show that the multiplexing gain of the sum-rate is $2M$. In section III we explore the effect of cross-over information on the multiplexing gain: we define and demonstrate that the multiplexing gain of the cognitive interference channel (where no cross-over messages are present) is 1, in contrast to the multiplexing gain of 2 seen in the cognitive X channel. These results allow us to compare the achievable rate regions of the cognitive and cognitive X channels in section IV at various SNRs. We conclude in section V. Due to space limitations, most of the proofs are deferred to [6] which may be found online.

II. THE MIMO COGNITIVE X CHANNEL

In this section we show that the MIMO cognitive X channel with M antennas at each of the 4 nodes has a multiplexing gain of $2M$, that of the broadcast channel of Fig. 1(3). The MIMO cognitive X channel is shown in Fig. 2(a). The word cognitive stems from the work [5] in which an interference channel with asymmetric side-information between Txs is called a *cognitive radio channel* (also known as an interference channel with degraded message sets in [13], [16]). In our figures, we denote side-information using a *double* arrow accompanied by the message that is known a priori. In the MIMO X -channel there are four messages: 11, 12, 21 and 22, which are encoded as M_{11}, M_{12}, M_{21} and M_{22} respectively. Our channel is a standard interference channel [2] with direct channel coefficients of 1 and cross-over coefficients of α_{12} and α_{21} . The M -dimensional transmitted random variables $X_1 \in \mathcal{X}_1$ and $X_2 \in \mathcal{X}_2$ are received as the signals $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$ in the sets according to the conditional distributions $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$. We consider an additive white Gaussian noise channel,

$$Y_1 = X_1 + \alpha_{21}X_2 + N_1 \quad (1)$$

$$Y_2 = \alpha_{12}X_1 + X_2 + N_2, \quad (2)$$

where $N_1 \sim \mathcal{N}(0, N_1)$ and $N_2 \sim \mathcal{N}(0, N_2)$ are independent and we assume individual average transmit power constraints of P_1 (P_2) on X_1 (X_2 resp.). The directed double arrow from X_1 to X_2 denotes *partial asymmetric side-information*: the encoding M_{11} is known to Tx 2.

Standard definitions of achievable rates and regions are employed [4], [5]. Although our achievable rate region will be defined for finite alphabet sets, in order to determine an achievable region for the Gaussian noise channel, specific forms of the random variables described in Thm. 1 are assumed. As in [3], [7], [9], Thm. 1 can readily be extended to memoryless channels with discrete time and continuous alphabets by finely quantizing the input, output, and interference variables (Gaussian in this case).

We now outline an achievable rate region for the Gaussian MIMO cognitive X -channel, which will be used to demonstrate a sum-rate scaling law of $2M$. The capacity region of the Gaussian MIMO broadcast channel [15] is achieved using Costa's dirty-paper coding techniques [3]. In the MIMO cognitive X -channel, at Tx 1, the encodings M_{11} and M_{12} may be jointly generated, for example using a dirty-paper like coding scheme. That is, one message may treat the other as non-causally known interference and code so as to mitigate it. At Tx 2, not only may the encodings M_{21} and M_{22} be jointly designed, but they may additionally use M_{11} as *a-priori* known interference. Thus, Tx 2 could encode M_{22} so as to potentially mitigate the interference Y_2 will experience from M_{11} as well as M_{21} . Let R_{11} be the rate from $M_{11} \rightarrow Y_1$, R_{12} from $M_{12} \rightarrow Y_2$, R_{21} from $M_{21} \rightarrow Y_1$ and R_{22} from $M_{22} \rightarrow Y_2$.

Theorem 1: Let $Z \triangleq (Y_1, Y_2, X_1, X_2, M_{11}, M_{12}, M_{21}, M_{22})$, and let \mathcal{P} be the set of distributions on Z that can

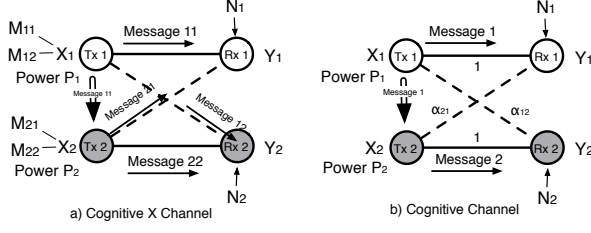


Fig. 2. Additive Gaussian noise interference channels with cross-over parameters α_{12}, α_{21} , transmitted encodings X_1, X_2 with expected transmit power limitations P_1 and P_2 , and received signals Y_1 and Y_2 . (a) Cognitive X channel: four messages encoded as $M_{11}, M_{12}, M_{21}, M_{22}$. M_{11} is the partial and asymmetric message knowledge at X_2 . (b) Cognitive channel: two messages encoded as M_{11}, M_{22} . X_1 is the asymmetric side-information known at X_2 .

be decomposed into the form

$$\begin{aligned} & p(m_{11}|m_{12})p(m_{12})p(m_{21})p(m_{22}|m_{11}, m_{21}) \\ & p(x_1|m_{11}, m_{12})p(x_2|m_{11}, m_{21}, m_{22}) \\ & p(y_1|x_1, x_2)p(y_2|x_1, x_2), \end{aligned} \quad (3)$$

where we additionally require $p(m_{12}, m_{22}) = p(m_{12})p(m_{22})$. For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all tuples $(R_{11}, R_{12}, R_{21}, R_{22})$ of non-negative real numbers such that:

$$\begin{aligned} R_{11} & \leq I(M_{11}; Y_1 | M_{21}) - I(M_{11}; M_{12}) \\ R_{21} & \leq I(M_{21}; Y_1 | M_{11}) \\ R_{11} + R_{21} & \leq I(M_{11}, M_{21}; Y_1) - I(M_{11}; M_{12}) \\ R_{12} & \leq I(M_{12}; Y_2 | M_{22}) \\ R_{22} & \leq I(M_{22}; Y_2 | M_{12}) - I(M_{22}; M_{11}, M_{21}) \\ R_{12} + R_{22} & \leq I(M_{12}, M_{22}; Y_2) - I(M_{22}; M_{11}, M_{21}) \end{aligned}$$

Any element in the closure of $\cup_{Z \in \mathcal{P}} S(Z)$ is achievable.

Proof: The codebook generation, encoding, decoding schemes and formal probability of error analysis are deferred to the manuscript in preparation [6] which may be found online. Heuristically, notice that the channel from $(M_{11}, M_{21}) \rightarrow Y_1$ is a multiple access which employs dirty paper coding [3], reducing the rate R_{11} by $I(M_{11}; M_{12})$ (like in Gel'fand-Pinsker [8] coding). Similarly, for the MAC $(M_{12}, M_{22}) \rightarrow Y_2$ the encodings M_{12} and M_{22} are independent (this is true in particular in the Gaussian case of interest in the next subsection, and so we simplify our theorem by ensuring the condition $p(m_{12}, m_{22}) = p(m_{12})p(m_{22})$) so that the regular MAC equations hold. The rate R_{22} is subject to a penalty of $I(M_{22}; M_{11}, M_{21})$ in order to guarantee finding an n -sequence m_{22} in the desired bin that is jointly typical with any given (m_{11}, m_{21}) . ■

A. MIMO Cognitive X-channel multiplexing gain is $2M$

We use the above achievable rate region to show that the sum-rate of the MIMO X-channel with *partial asymmetric side-information* has a multiplexing gain $2M$.

Corollary 2: Consider the MIMO additive Gaussian X-channel with partial asymmetric side-information described in eqns. (1), (2) and Fig. 2(a) with $P_1 = P_2 = P$. Then

$$\lim_{P \rightarrow \infty} \frac{\max R_{11} + R_{12} + R_{21} + R_{22}}{\log P} = 2M, \quad (4)$$

where the max is taken over all $(R_{11}, R_{12}, R_{21}, R_{22}) \in \mathcal{C}_{X-cog}$, where \mathcal{C}_{X-cog} is the capacity region of the MIMO cognitive X-channel.

Proof: We sketch the proof for $M = 1$, and defer details, as well as the more involved proof for general M to [6]. Roughly speaking, the general M case is proven by evaluating the same mutual information terms of Thm. 1, as done for $M = 1$, with the added complications (such as matrix inversions) that arise from considering vectors rather than scalars.

First, note that the multiplexing gain of the MIMO broadcast channel, whose capacity region outer bounds ours, with 2 transmit antennas and single receive antennas at the Rxs is 2. We will in fact prove that 2 is achievable using the scheme of Thm. 1. To do so, we specify forms for the variables, and then optimize the dirty paper coding parameters, similar to Costa's technique [3]. The Gaussian distributions we assume on all variables are of the form

$$\begin{aligned} M_{11} &= U_{11} + \gamma_1 U_{12} & U_{11} &\sim \mathcal{N}(0, P_{11}) \\ M_{12} &= U_{12}, & U_{12} &\sim \mathcal{N}(0, P_{12}) \\ M_{21} &= U_{21}, & U_{21} &\sim \mathcal{N}(0, P_{21}) \\ M_{22} &= U_{22} + \gamma_2 (U_{21} + a_{12} U_{11}) & U_{22} &\sim \mathcal{N}(0, P_{22}) \end{aligned}$$

$$X_1 = U_{11} + U_{12}$$

$$X_2 = U_{21} + U_{22} + \sqrt{\frac{(1-\beta)P_2}{P_{11}}} U_{11}$$

$$Y_1 = \left(1 + a_{21} \sqrt{\frac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + U_{12} + a_{21}(U_{21} + U_{22}) + N_1$$

$$Y_2 = \left(a_{12} + \sqrt{\frac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + a_{12} U_{12} + (U_{21} + U_{22}) + N_2.$$

where $P_1 = P_{11} + P_{12}$ and $\beta P_2 = P_{21} + P_{22}$.

Here the variables $U_{11}, U_{12}, U_{21}, U_{22}$ are all independent, encoding the four messages to be transmitted. The β parameter divides power at Tx 2: βP_2 is used in transmitting its own messages, m_{21} and m_{22} , while the remainder of the power, $(1 - \beta)P_2$ is used to reinforce the message encoded as m_{11} of Tx 1. Notice that because we only assume partial message knowledge (only *one* of the two messages of Tx 1 is known at Tx 2), only one of these messages gets reinforced at Tx 2. Also note that $p(m_{11}, m_{21}) = p(m_{11})p(m_{21})$ and $p(m_{12}, m_{22}) = p(m_{12})p(m_{22})$ as needed in Thm. 1. The rates $R_1 = R_{11} + R_{21}$ and $R_2 = R_{12} + R_{22}$ to each Rx can be calculated separately. Each can be maximized with respect to the relevant dirty-paper coding parameter (γ_1 for Tx 1, and γ_2 for Tx 2). The bounds of Thm. 1 may be evaluated by combining the appropriate determinants of sub-matrices of the overall covariance matrix $E[\Theta\Theta^T]$ where $\Theta \triangleq (M_{11}, M_{21}, M_{12}, M_{22}, Y_1, Y_2)$. The six bounds of Thm. 1 are evaluated explicitly in [6] online; we simply demonstrate the resulting expressions for the sum-rate to Rx 1, R_1 as well as to Rx 2, R_2 in the expressions (5) and (6) resp. In order to simplify the expressions, we have set

$$\theta \triangleq a_{21} \sqrt{\frac{(1-\beta)P_2}{P_{11}}} \quad \text{and} \quad \eta \triangleq (a_{12} + \theta).$$

$$R_{11} + R_{21} \leq \frac{1}{2} \log_2 \left(\frac{P_{11}(P_{11}(1+\theta)^2 + P_{12} + \alpha_{21}^2(P_{21} + P_{22}) + N_1)}{\gamma_1^2(P_{12}(P_{11} + \alpha_{21}^2 P_{22} + N_1)) - 2\gamma_1 P_{11} P_{12} + P_{11}(P_{12} + \alpha_{21}^2 P_{22} + N_1)} \right) \quad (5)$$

$$R_{12} + R_{22} \leq \frac{1}{2} \log_2 \left(\frac{(P_{11}\eta^2 + \alpha_{12}^2 P_{12} + P_{21} + P_{22} + N_2)(\gamma_1^2 P_{12}(P_{22} + \gamma_2^2 \eta^2 P_{11}) + P_{11} P_{22})}{(P_{11} + \gamma_1^2 P_{12})\gamma_2^2(P_{11}\eta^2 + P_{21})(P_{22} + N_2) + \gamma_2(-2P_{22}(P_{11}\eta^2 + P_{21}) + P_{22}(P_{11}\eta^2 + P_{21} + N_2))} \right) \quad (6)$$

We could search for the γ_1 and γ_2 that jointly optimize $R_1 + R_2$. However, noticing that R_1 depends only on γ_1 , we heuristically select γ_1 to maximize R_1 , as

$$\gamma_1 = \frac{P_{11}(1+\theta)}{P_{11}(1+\theta)^2 + \alpha_{21}^2 P_{22} + N_1}. \quad (7)$$

When we substitute this γ_1 into the bounds on Rx 1's total sum-rate we obtain the bound (8). Notice that an important cancellation occurs in the denominator of (8) when the optimal γ_1 is substituted in the sum-rate expression.

To maximize the R_1 bound, we had set γ_1 as in (7). Although we could maximize R_2 with respect to γ_2 , we use a simpler and more heuristic approach and simply minimize the denominator of the sum-rate $R_2 = R_{12} + R_{22}$ with respect to γ_2 , which yields $\gamma_2 = \frac{P_{22}}{P_{22} + N_2}$. It is interesting to note that this is exactly the same dirty paper coding parameter as Costa derives. It is intuitively pleasing, and although it does not strictly maximize R_2 with respect to γ_2 , as we will see shortly, it performs sufficiently well in the limit of large SNR, thus performs asymptotically optimally. If we fix P_{22} (does not scale with P) and let $P_{11} = P_{12} = P_{21}$ all scale like P , subject to $P_{11} + P_{12} = P$ and $P_{21} + P_{22} = P$, then the bound on the total sum rate to both Rx's $R_{11} + R_{12} + R_{21} + R_{22}$ scales like $2 \log P$. This can be seen by noting that γ_2 remains fixed and $\gamma_1 \rightarrow 1$ as $P \rightarrow \infty$. Keeping P_{22} fixed was crucial for achieving the $\log P$ scaling in R_1 . Intuitively, this is because of the asymmetric message knowledge; the interference Tx 2 causes the Tx 1 is not mitigated. Keeping P_{22} constant still allows Tx 2 to dirty paper code, or mitigate the interference caused by M_{11} and M_{21} to Rx 2's signal Y_2 , while causing asymptotically (as $P_{11}, P_{12}, P_{21} \rightarrow \infty$) negligible interference to Y_1 . We note that the sum-rate scaling does not depend on the power division parameter β . That is, the sum-rate scaling remains 2 for any $\beta > 0$ (meaning some power is in fact allocated to transmit the messages of Tx 2, and this power will then $\rightarrow \infty$). The joint optimization of γ_1, γ_2 could lead to higher rates in lower SNR regimes. However, at high SNR our choice of parameters leads to an optimal sum-rate scaling. ■

III. THE COGNITIVE INTERFERENCE CHANNEL

In the previous section we demonstrated that the scaling law of the sum-rate of the MIMO cognitive X channel, with *partial transmitter side-information* is $2M$, which is optimal in the limit as $\text{SNR} \rightarrow \infty$. In this section

we investigate whether partial asymmetric side-information is always equivalent to full symmetric transmitter side information in terms of sum-rate scaling as $\text{SNR} \rightarrow \infty$. To do so we look at another channel with partial asymmetric side information at the transmitters: the recently explored cognitive interference channel (also known as the interference channel with degraded message sets [13] or the cognitive radio channel [5]), shown in Fig. 2(b). We consider the same additive Gaussian noise channel as in (1), (2). The only difference with the cognitive X -channel is the absence of cross-over messages 12 and 21. We will see that while partial asymmetric side information in the X -channel results in the same sum-rate scaling as a fully cooperative (at the transmitters) X -channel, the opposite is true of partial asymmetric side information in the interference channel: at high SNR its sum-rate scales like the interference channel. In other words, although partial side-information may help the interference channel in a medium SNR-regime [5], [13], at high SNR, one cannot improve the scaling law of the sum-rate. The Gaussian cognitive interference channel considered here is the same channel as that of [13], where its capacity region is derived for the case $\alpha_{21} \leq 1$, and sum-rate capacity is found for $\alpha_{21} > 1$. The next theorem is a direct result from this capacity region.

Theorem 3: Consider the Gaussian interference channel where Tx 2 has non-causal knowledge of the message of Tx 1, described in eqns. (1), (2) and Fig. 2(b) with $P_1 = P_2 = P$. Then

$$\lim_{P \rightarrow \infty} \frac{\max_{(R_1, R_2) \in \mathcal{C}} R_1 + R_2}{\log P} = 1, \quad (9)$$

where R_i corresponds to the rates from the i -th source to the i -th Rx, and \mathcal{C} is the capacity region of the channel. □

The proof of this result is deferred to work [6] which may be viewed online. It employs eqns. (24), (25) and Corollary 4.1 of [13]. This theorem shows that the cognitive channel of Fig. 2(b) behaves like the interference channel at high SNR. Thus, when partial side-information is available at the Txs, although achievable rates may be enhanced in the low and medium SNR range as compared to the interference channel, at large SNR, the two channels scale in the same manner. This is in sharp contrast to the cognitive X channel, where partial asymmetric side information results in the same sum-rate scaling as the fully cooperative broadcast channel at high SNR.

$$R_{11} + R_{21} \leq \frac{1}{2} \log_2 \left(\frac{(P_{11}(1+\theta)^2 + P_{12} + \alpha_{21}^2(P_{21} + P_{22}) + N_1)(P_{11}(1+\theta)^2 + \alpha_{21}^2 P_{22} + N_1)}{(\alpha_{21}^2 P_{22} + N_1)(P_{11}(1+\theta)^2 + P_{12} + \alpha_{21}^2 P_{22} + N_1)} \right) \quad (8)$$

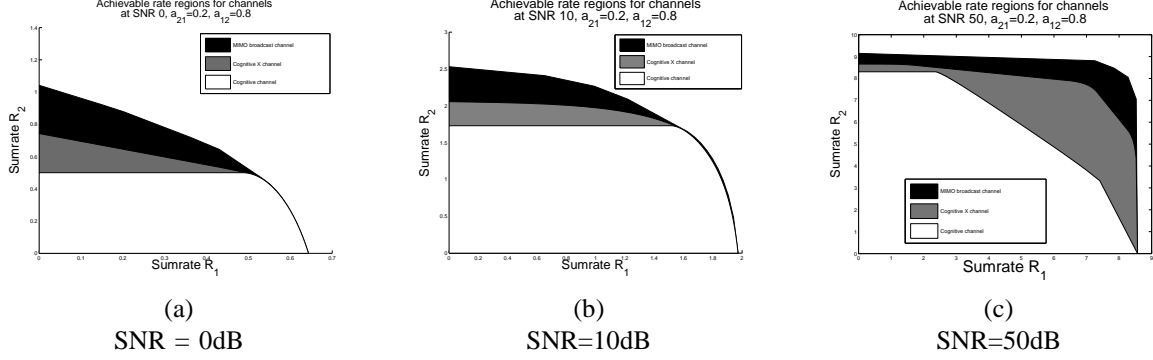


Fig. 3. Comparison of the cognitive interference and the cognitive X channels at various SNRs.

IV. COMPARISON OF COGNITIVE AND COGNITIVE X CHANNEL REGIONS AT VARIOUS SNRS

In this section, we numerically evaluate the capacity region of the cognitive channel of Fig. 2(b) and [13] and compare it with the achievable region of the cognitive X channel described in Thm. 1 and Fig. 2(a) under our choice of variables, as well as the MIMO broadcast channel with 2 Tx antennas and 2 single antenna receivers. In doing so, we highlight the dependence of the rate region, and in particular the sum-rate scaling, on the SNR. At 0 and 10dB receive SNR, the two regions almost coincide for large R_1 . At high SNR (50dB) the sum-rate scaling increases, and the gap between the sum-rate achieved by the cognitive interference and the cognitive X-channels widens, confirming the sum-rate scaling laws of 1 and 2 respectively. Fig. 3 contrasts the achievable rate regions for cross-over parameters $\alpha_{12} = 0.8$, $\alpha_{21} = 0.2$ at the SNRs 0 and 50 dB. Notice that the broadcast channel always forms an outer bound, but that the achievable rate region appears to be tight as for large R_1 . We speculate the achievable rate region is tight for areas of the rate region where the slope of the tangent is smaller than -1).

V. CONCLUSION

In this paper we have shown that the multiplexing gain of the sum-rate of the MIMO cognitive X channel is $2M$, achieving the optimal sum-rate scaling. Particularly surprising, and pleasing, is that the optimal sum-rate scaling may be achieved with only partial asymmetric side-information at the Txs. This could lead one to think that partial asymmetric side-information, rather than full side-information (or cooperation) between Txs always yields the optimal sum-rate scaling. However, we then showed that this is not the case: in the interference channel with asymmetric side-information, where no cross-over information is permitted, the scaling of the sum-rate for the single antenna channel is 1, rather than the optimal 2. This demonstrates that at large SNR, asymmetric cooperation

does not give any gain, in terms of scaling, over an interference channel in which there is no Tx cooperation.

REFERENCES

- [1] G. Caire and S. Shamai, "On the achievable throughput of a multi-antenna gaussian broadcast channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691–1705, July 2003.
- [2] A. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, vol. IT-24, no. 1, pp. 60–70, Jan. 1978.
- [3] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. IT-29, pp. 439–441, May 1983.
- [4] T. Cover and J. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons, 1991.
- [5] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.
- [6] N. Devroye and M. Sharif, "The multiplexing gain of 2×2 channels with ideal transmitter cooperation." [Online]. Available: <http://www.deas.harvard.edu/~ndevroye/research/devroyesharif2x2.pdf>
- [7] R. G. Gallager, *Information Theory and Reliable Communication*. New York: Wiley, 1968, ch. 7.
- [8] S. Gel'fand and M. Pinsker, "Coding for channels with random parameters," *Probl. Contr. and Inf. Theory*, vol. 9, no. 1, pp. 19–31, 1980.
- [9] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 1, pp. 49–60, 1981.
- [10] A. Host-Madsen and A. Nosratinia, "The multiplexing gain of wireless networks," in *2005 IEEE International Symposium on Information Theory*, Sept. 2005.
- [11] S. A. Jafar, "Degrees of freedom on the MIMO X channel - optimality of zero forcing and the MMK scheme." [Online]. Available: <http://www.ece.uci.edu/~syed/resume.html>
- [12] S. A. Jafar and M. Fakhreddin, "Degrees of freedom for the MIMO interference channel," in *2006 IEEE International Symposium on Information Theory*, July 2006.
- [13] A. Jovicic and P. Viswanath, "Cognitive radio: An information-theoretic perspective," Submitted to *IEEE Trans. Inf. Theory*, 2006.
- [14] M. Maddah-Ali, A. Motahari, and A. Khandani, "Signaling over MIMO multi-base systems: combination of multi-access and broadcast schemes," in *2006 IEEE International Symposium on Information Theory*, July 2006.
- [15] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," Submitted to *IEEE Trans. Inf. Theory*, July 2004.
- [16] W. Wu, S. Vishwanath, and A. Aripistathis, "On the capacity of interference channel with degraded message sets," Submitted to *IEEE Trans. Inf. Theory*, 2006.